

# HEAVY MESON DYNAMICS IN A QCD RELATIVISTIC POTENTIAL MODEL

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## ABSTRACT

We use a QCD relativistic potential model to compute the strong coupling constant  $g$  appearing in the effective Lagrangian which describes the interaction of  $0^-$  and  $1^-$   $\bar{q}Q$  states with soft pions in the limit  $m_Q \rightarrow \infty$ . We compare our results with other approaches; in particular, in the non relativistic limit, we are able to reproduce the constituent quark model result:  $g = 1$ , while the inclusion of relativistic effects due to the light quark gives  $g = \frac{1}{3}$ , in agreement with QCD sum rules. We also estimate heavy meson radiative decay rates, with results in agreement with available experimental data.

## 1. The Strong Coupling Constant $g_{D^*D\pi}$

The decay  $D^{*+} \rightarrow D^0\pi^+$  is described in terms of a strong coupling constant  $g_{D^*D\pi}$  defined by:  $\langle D^0(k)\pi^+(q)|D^{*+}(p,\epsilon) \rangle = g_{D^*D\pi} \epsilon^\mu q_\mu$ . CLEO collaboration measurement <sup>1</sup>:  $BR(D^{*+} \rightarrow D^0\pi^+) \simeq 68.1 \pm 1.0 \pm 1.3\%$  and the upper bound<sup>2</sup> :  $\Gamma(D^{*+}) < 131 \text{ KeV}$  provide us with the constraint :  $g_{D^*D\pi} < 20.6$ .

The interest in the evaluation of this coupling constant is manifold. The form factor  $F_1(q^2)$  describing the semileptonic decay  $B \rightarrow \pi\ell\nu$  is believed to be dominated by the  $B^*$  pole, so that its value at maximum transferred momentum is proportional to  $g_{B^*B\pi}$ , which is related to  $g_{D^*D\pi}$  by <sup>3</sup>:  $g_{P^*P\pi} = 2 m_P g/f_\pi$ , where  $P(P^*)$  is a  $0^-$  ( $1^-$ ) heavy meson of mass  $m_P$ , and  $g$  is independent of  $m_P$ . Besides,  $g$  appears in the effective Lagrangian describing the interaction between heavy mesons and light Nambu-Goldstone bosons <sup>4,5,6</sup>.

In non relativistic quark models <sup>6,7</sup>  $g \simeq 1$ , while recent QCD sum rules <sup>8</sup> and HQET <sup>9,10</sup> analyses give:  $g \simeq 0.2 - 0.4$ . We wish to show that the inclusion of the relativistic effects in the bound state can lower the value  $g = 1$ , providing an explanation of the discrepancy between the different approaches.

We shall obtain  $g$  in the framework of a QCD relativistic potential model <sup>11</sup>. In this model the  $\bar{q}Q$  heavy states  $D$  and  $D^*$  are described in terms of the creation operators of the constituent quarks and of a meson wave function  $\psi$ , normalized according to:  $\frac{1}{(2\pi)^3} \int d\vec{k} |\psi|^2 = 2\sqrt{m_D^2 + \vec{p}^2}$ , where  $\vec{p}$  is the meson momentum and  $\vec{k}$  is the quark relative momentum.  $\psi$  satisfies a Salpeter equation\* which includes

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\*This equation arises from the bound-state Bethe Salpeter equation by considering the instantaneous approximation and restricting the Fock space to the  $\bar{q}Q$  pairs <sup>12</sup>.

relativistic effects in the quark kinematics. In this equation the interquark potential  $V$  coincides, in the meson rest frame, with the Richardson potential, with a linear behaviour for large distances, reproducing QCD confinement, and a coulombic shape for small distances, with the further assumption that  $V$  is constant near the origin in order to avoid unphysical singularities. The values of the quark masses, obtained by fits to meson masses, are:  $m_u = m_d = 38 \text{ MeV}$ ,  $m_s = 115 \text{ MeV}$ ,  $m_c = 1452 \text{ MeV}$ ,  $m_b = 4890 \text{ MeV}$ .

To evaluate  $g_{D^*D\pi}$  let us consider the matrix element of the axial current  $A_\mu$  between the states  $D^*$  and  $D$ :

$$\begin{aligned} \langle D^0(k) | A_\mu | D^{*+}(p, \epsilon) \rangle = & -i \left\{ \epsilon_\mu (m_{D^*} + m_D) A_1(q^2) - \frac{\epsilon \cdot q}{m_D + m_{D^*}} (p + k)_\mu A_2(q^2) \right. \\ & \left. - \frac{\epsilon \cdot q}{q^2} 2m_{D^*} q_\mu [A_3(q^2) - A_0(q^2)] \right\}, \end{aligned} \quad (1)$$

where  $2m_{D^*} A_3 = (m_D + m_{D^*}) A_1 + (m_{D^*} - m_D) A_2$  ( $q = p - k$ ).

Taking the derivative of  $A_\mu$ , we can link the l.h.s. of Eq. (1) to the matrix element of the pseudoscalar current between the same states, which is supposed to be dominated by the  $\pi^+$  pole. Performing the overlap of the states  $D$ ,  $D^*$  and the axial current, we have in the chiral limit:  $g_{D^*D\pi} = \frac{2m_{D^*}}{f_\pi} A_0(0)$ , and finally:

$$g = A_0(0) = \frac{1}{4m_D} \int_0^\infty dk |\tilde{u}(k)|^2 \frac{E_q + m_q}{E_q} \left[ 1 - \frac{k^2}{3(E_q + m_q)^2} \right], \quad (2)$$

where  $E_q = \sqrt{k^2 + m_q^2}$  and  $\tilde{u}(k) = \frac{k \psi(k)}{\sqrt{2\pi}}$ . The wave functions  $\tilde{u}(k)$  come from a numerical solution of the Salpeter equation. We obtain <sup>11</sup>:  $A_0(0) = 0.4$  ( $D$  case);  $A_0(0) = 0.39$  ( $B$  case), giving so:  $g_{D^*D\pi} = 12.3$  and  $g_{B^*B\pi} = 31.7$ . We can observe that we have only 2% deviation from the scaling result  $g_{D^*D\pi}/g_{B^*B\pi} = m_D/m_B$ .

It is interesting to notice that, in the non relativistic limit, i.e.  $E_q \simeq m_q \gg k$ , we obtain:  $g = \frac{1}{2m_D} \int_0^\infty dk |\tilde{u}(k)|^2 = 1$ , reproducing the constituent quark model result. On the other hand, in the limit  $m_q \rightarrow 0$ ,  $m_Q \rightarrow \infty$ , the result is:  $g = 1/3$ , in agreement with the QCD sum rules determination.

## 2. Radiative Heavy Meson Decays

The evaluation of radiative decay rates involves the knowledge of the matrix element of the electromagnetic current  $J_\mu^{e.m.}$  between the states  $D^*$  and  $D$ :

$$\langle D^+(k) | J_\mu^{e.m.} | D^{*+}(p, \epsilon) \rangle = \left( \frac{e_Q}{\Lambda_Q} + \frac{e_q}{\Lambda_q} \right) \epsilon_{\mu\nu\alpha\beta} \epsilon^\nu k^\alpha p^\beta, \quad (3)$$

where  $e_Q$  ( $e_q$ ) is the heavy (light) quark electric charge. In the framework of the relativistic QCD model we are able to evaluate the "effective" masses  $\Lambda_q$ ,  $\Lambda_Q$  by computing the overlap of meson states and the electromagnetic current. We find

Table 1. Heavy meson radiative decay rates

Decay rate/ BR	theory	experiment
$\Gamma(D^{*+})$	46.21 <i>KeV</i>	< 131 <i>KeV</i>
$BR(D^{*+} \rightarrow D^+\pi^0)$	31.3%	$30.8 \pm 0.4 \pm 0.8\%$
$BR(D^{*+} \rightarrow D^0\pi^+)$	67.7%	$68.1 \pm 1.0 \pm 1.3\%$
$BR(D^{*+} \rightarrow D^+\gamma)$	1.0%	$1.1 \pm 1.4 \pm 1.6\%$
$\Gamma(D^{*0})$	41.6 <i>KeV</i>	
$BR(D^{*0} \rightarrow D^0\pi^0)$	50.0%	$63.6 \pm 2.3 \pm 3.3\%$
$BR(D^{*0} \rightarrow D^0\gamma)$	50.0%	$36.4 \pm 2.3 \pm 3.3\%$
$\Gamma(D_s^*) = \Gamma(D_s^* \rightarrow D_s\gamma)$	0.382 <i>KeV</i>	
$\Gamma(B^{*-}) = \Gamma(B^{*-} \rightarrow B^-\gamma)$	0.243 <i>KeV</i>	
$\Gamma(B^{*0}) = \Gamma(B^{*0} \rightarrow B^0\gamma)$	$9.2 \cdot 10^{-2}$ <i>KeV</i>	
$\Gamma(B_s^*) = \Gamma(B_s^* \rightarrow B_s\gamma)$	$8.0 \cdot 10^{-2}$ <i>KeV</i>	

$\Lambda_c \simeq m_c = 1.57$  *GeV*,  $\Lambda_b \simeq m_b = 4.95$  *GeV* and  $\Lambda_q = 0.48$  *GeV*  $\gg m_q$ . The B.R. are reported in Table I <sup>11</sup> together with the available experimental data.

We conclude that the inclusion of relativistic effects in a QCD potential model has allowed us to explain the discrepancy among different models in the evaluation of the heavy meson coupling constant with soft pions. Moreover, the same model, applied to heavy meson radiative transitions, gives results in agreement with experimental data.

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